FREQUENCY-DOMAIN METHODS FOR EXTENDING TDR MEASUREMENT RANGE IN SALINE SOILS

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ABSTRACT

Water content and electrical conductivity of soils are routinely determined using time-domain reflectometry (TDR) based on analysis of signal travel time along buried waveguides. In soils with appreciable electrical conductivity, travel time analysis becomes progressively inaccurate due to signal attenuation to the point of failure (typically at EC_a > 2 dS m^{-1}). We demonstrate that information on bulk dielectric permittivity, lost in travel time analysis in saline soils, can be recovered in the frequency domain by using shorter waveguides to reduce signal attenuation. We implement a robust algorithm for the time-to-frequency domain transformation (Nicolson, 1973) that does not rely on waveform differentiation. This algorithm uses signal de-ramping to eliminate noise induced by standard differentiation and to reduce truncation errors arising from finite sample size. The methodology was tested using coaxial cells and three-wire TDR probes under a wide range of solution electrical conductivities (0 to 24 dS m^{-1}). Reliable estimates of bulk dielectric permittivity for EC’s up to 24 dS m^{-1} in a silt loam soil were obtained using 2 and 3 cm TDR probes. In addition to bulk dielectric permittivity, this method provides additional frequency-dependent and electrical conductivity information derived in the estimation of the Cole-Cole parameters.

INTRODUCTION

Time Domain Reflectometry (TDR) offers simultaneous and accurate method for water content and electrical conductivity determination in soils and other porous media. Waveform reflections necessary for permittivity measurements can be totally attenuated in lossy (highly saline) materials. Soil texture, salinity, probe geometry and water content all influence permittivity measurements. Nadler et al. (1999) found that under field capacity in sandy and loamy soils, TDR could be safely used up to an apparent electrical conductivity, EC_a, of approximately 2 dS m^{-1}. TDR applications are therefore limited to soils with relatively low salinity unless measures to preserve waveform reflection from the end of a waveguide are taken. Rod coating methods have been successfully used to reduce signal attenuation and preserve permittivity information in highly saline soils. Since these coatings significantly influence the resulting permittivity, rod-coating calibration is required (Mojid et al., 1998) making this a less appealing method.

Time-domain methods employ a step voltage which propagates down a low-loss coaxial line and whose voltage remains relatively constant until a change in material is encountered. At the interface of two materials with different permittivities, a portion of the signal is reflected and the remainder is transmitted through the new material. In lossy materials, the presence
of free ions give rise to attenuation of the transmitted signal along the travel path due to ion migration between rods of different polarity. For material samples of finite length, a reflection occurs at the sample-air interface, resulting in a chain of signal reflections as the new reflected signal again reflects and transmits at the sample-cable interface resulting in multiple reflections shown in Fig 1.

Conversion of TDR waveforms into the frequency domain provides additional frequency-dependent dielectric information (Heimovaara, 1994; Friel and Or, 1999). Information on electrical conductivity, relaxation frequency, static and apparent permittivity may be extracted using this procedure. The disadvantages of this approach are that the process including fast Fourier transformation (FFT) of the waveform and fitting of an appropriate model to the transformed scatter function can be laborious. The procedure, however, has the potential to be automated to make it more amenable to realtime measurements.

The objectives of this work address both technical and theoretical aspects, the former objectives were to determine the extent to which dielectric and electrical information in saline porous media can be extracted by a) transforming TDR waveforms to the frequency domain via Fourier analysis, and b) using shorter probes to reduce attenuation. The latter objectives were to improve time to frequency transformation by a) introducing and testing the Nicolson (1973) ramping algorithm as a substitute for waveform differentiation and b) implementing a more robust system input function for deriving the scatter function.

**THEORETICAL**

Transformation of the measured and input signal waveforms from the time to the frequency domain yields the system response or the so-called scatter function that represents interactions between the material (dielectric permittivity) and the electric field in a given waveguide geometry. Hence, for a given geometry (coaxial in this study), The transformed scatter function may be used to infer the material dielectric properties using the Debye (1929) or Cole-Cole (1941) models for dielectric permittivity.

As a TDR input signal, \( v(t) \), traverses the waveguide, the response function, \( r(t) \), reflects interactions between the applied EM field and the sample. These interactions are summarized in the waveform, \( r(t) \), described by the following convolution integral (van Gemert, 1973)

\[
r(t) = \int_{-\infty}^{t} v_{0}(t - \tau) s(\tau) d\tau
\]

where \( t \) is the time lag variable of integration and \( s(\tau) \) is dependent upon the probe geometry and the material properties. Application of the convolution theorem reduces this integral to a simple product in the frequency domain. The Fourier transforms of the frequency-dependent response, \( R(f) \), system, \( S_{11}(f) \), and input, \( V_{0}(f) \) functions described by (Lathi, 1992)

\[
R(f) = V_{0}(f) S_{11}(f)
\]

where \( f \) [Hz] is the frequency. In general the Fourier transform is written as
\[
X(f) = \int_{-\infty}^{\infty} x(t)e^{i(-2\pi ft)} dt
\]

where \(i = (-1)^{\frac{1}{2}}\) and \(r(t)\) and \(v(t)\) are substituted for \(x(t)\).

The measurement of reflections along a transmission line is a common method used in both time and frequency domain spectroscopy. In each case the signal is analyzed similarly with only the dependent variable differing (i.e., time or frequency). The multiple reflections in a coaxial transmission line can be modeled according to the scatter function given by Clarkson et al. (1977) as

\[
S_{tt}(f) = \frac{\rho + \epsilon e^{-2\gamma L}}{1 + \rho \cdot \epsilon e^{-2\gamma L}}
\]

where \(\rho\) is the reflection coefficient described as

\[
\rho = \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}}
\]

in which \(\epsilon\) is the impedance ratio of the cable, \(z_c\), and probe, \(z_p\), (i.e., \(\epsilon = z_c/z_p\)), \(\epsilon^*(f)\) is the complex dielectric permittivity and \(\gamma L\) is the TEM mode propagation constant written as

\[
\gamma L = \frac{i2\pi fL[\epsilon^*(f)]^{0.5}}{c}
\]

in which \(L\) [m] is the probe length and \(c\) is the speed of light constant (3 x 10^8 m s^{-1}).

The frequency-dependent complex dielectric constant, \(\epsilon^*(f)\), may be described by the following parametric model of Debye (1929) as modified by Cole and Cole (1941), written as

\[
\epsilon^*(f) = \left[ \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + \left( \frac{f}{f_{\text{rel}}} \right)^{(1-\beta)}} \right] - i\frac{\sigma_{\text{dc}}}{2\pi \epsilon_0 f}
\]

where \(\epsilon_0\) is the static dielectric permittivity, \(\epsilon_s\) is the permittivity at infinite frequency, \(f_{\text{rel}}\) is the dielectric relaxation frequency of the material, \(\sigma_{\text{DC}}\) is the low frequency electrical conductivity [S m^{-1}], \(\epsilon_\infty\) is the permittivity of free space (8.854 x 10^{-12} F m^{-1}) and \(\beta\) is a parameter added by Cole and Cole (1941) to describe the spread in relaxation frequency which tends to increase as the complexity of the mixture increases (e.g., minerals, biological materials). For pure liquids with a single relaxation frequency such as water or ethanol, \(\beta\) is zero resulting in the original debye model.

**EXPERIMENTAL**

A Tektronix 1502B cable tester was used to obtain time domain reflectometry (TDR) waveforms. Waveforms were collected and analyzed using WINTDR99, an analysis software package (Or et al., 1999) available at
http://psb.usu.edu/wintdr99/index.html. The maximum sampling frequency was 14.9 GHz (distance/division = 0.25 m) and waveforms consisted of 2510 data points. Both coaxial (length = 19, 6, and 2 cm) and 3-wire (Length = 2, 3, 6, 10, and 15 cm) TDR probes (Fig. 2) were used for measurements in solutions and a saturated sand and in saturated Millville Silt Loam soil. Distilled water, Ethanol and Octanol-1 were used as solutions in the coaxial cell maintained at 25°C. Potassium chloride solutions of 0, 3, 6, 12, 18, 24 dS m\(^{-1}\) in saturated sand and solutions of 36 and 48 dS m\(^{-1}\) were also used in a saturated Millville silt loam soil.

**AUTOMATION CONSIDERATIONS**

**Waveform collection**

The size of the waveform used for frequency domain analysis is a function of the sampling frequency (distance per division per screen) and the total number of consecutive screens which are collected, each screen or window from the Tektronix contains 251 data points. The combination of sampling frequency and number of screens or windows collected will determine the frequency content obtained in the frequency domain. The available frequency content in time domain analysis was found by Heimovarra (1994) to lie in the range 20 kHz to 1.5 GHz. Automated waveform collection may be performed using TDR analysis software such as WINTDR99 whose output from each measurement may then be combined to form a waveform of desired length. For example, we collected 10 consecutive screens yielding a waveform of 2510 data points. The time (\(t\)) or distance (\(x\)) between two consecutive points in the waveform are computed to find the maximum sampling frequency. With a distance per division (\(dd\)) setting of 0.25 m, and assuming a velocity of propagation, \(V_p\), equal to 0.99, we calculate the maximum sampling frequency (\(f_{\text{max}}\)) from

\[
f_{\text{max}} = \frac{c V_p}{2 \cdot \Delta t} = \frac{c V_p}{2 \cdot \Delta x} = \frac{251 \cdot c V_p}{2 \cdot \Delta d \cdot \Delta x}
\]

(8)

where \(c\) is the speed of light, the number of divisions per screen (\(ds\)) was 10 for the Tektronix device and the division by 2 accounts for the doubling of the travel distance in the reflection measurement. For the conditions described above, \(f_{\text{max}} = 15\) GHz based on \(V_p\) of 0.99. The lowest frequency content, including all points in the waveform, can be calculated using eq. (8), as \(f_{\text{min}} = c V_p / (2 \cdot \pi \cdot N)\), where \(N = 2510\) data points, giving \(f_{\text{min}} = 6\) MHz.

**Input function derivation**

The input function provides a way of removing the unwanted noise caused by the cable and impedance transition to the probe. The input function can be obtained by removing one of the conductors from the probe and obtaining the waveform shown in Fig. 3. Removal of one of the conductors may not be possible, especially for pre-manufactured TDR probes. An alternate approach is to generate an artificial input function, which requires only the measurement of the waveform with the probe in air and an algorithm to determine the initial slope of the waveform and the reflection coefficient at the distal point shown in Fig3. The resulting artificially generate waveform shown in Fig 3,

**Figure 2.** Coaxial and three-wire TDR probes, of varying length, used for permittivity and electrical conductivity measurements in different media.

**Figure 3.** Waveforms derived from TDR measurements using a 3-wire probe in air, with the central conductor removed and an artificially generated waveform. The input function may be represented by the waveform with the central conductor removed or, as an alternative, it may be generated based on the initial slope and final value of a waveform measured in air.
has another advantage which is to avoid the secondary reflections of the waveform where the central conductor was removed from the probe.

**Waveform preparation and transformation**

Transformation of a discrete waveform, derived in the time domain, to the frequency domain is accomplished using a discrete Fourier transformation (DFT) technique. The fast Fourier transform (FFT) is commonly employed for this purpose and is generally included as a function in modern mathematical software programs. Simply taking the FFT of the waveform from TDR is problematic in that the last point of the waveform is generally non-zero and introduces frequency errors. Heimovaara (1994) used the backward difference of the waveform with the assumption that zeroes result at the beginning and end of the data. Noise enhancement introduced when taking the derivative of two similar quantities in digital systems can be significant and therefore Nicolson (1973) suggested simply subtracting a ramp from the step response as illustrated in Fig. 4. The linear ramp is subtracted from the original waveform \( W(n) \) at each point, \( n (0<n<N) \), and is scaled according to the final point \( N \). The modified waveform, \( W'(n) \), ensures a final value of zero, written as

\[
W'(n) = W(n)
\]

A key feature of this algorithm is that the DFT of \( W^*(n) \) produces the same response as an infinite train of samples truncated out, so for discrete frequencies, \( \omega \), where \( \omega = (2\pi f_s)n/N \), there is no error, which is not the case for \( \omega = (2\pi f_s)n/N \) (Nicolson, 1973).

After applying the Nicolson ramping algorithm or differentiating each waveform (both input and response functions), they require further preparation prior to applying the discrete fast Fourier transform (FFT). The total number of data points in the final waveform, prior to Fourier transformation should be greater than or equal to the number of points in the original measured waveform, \( N \), and equal to \( 2^k \) where \( k \) is an integer. For the case of 2510 data points, which lies between \( 2^{11} = 2048 \) and \( 2^{12} = 4096 \), we ultimately require 4096 data points and therefore pad the final waveform between 2510 and 4096 data points using zeros.

After zero padding, the waveform is transformed using FFT, a function in most mathematical or spreadsheet software. The scatter function, \( S_{11}(f) \), is obtained by dividing the transformed response function, \( R(f) \), by the transformed input function, \( V_o(f) \), from eq. (2). Transformed scatter functions plotted in Fig. 5, derived from TDR measurements in three liquids of known permittivities, illustrate the dependence of the scatter function’s characteristic period on dielectric permittivity. Note a distinct change in the \( S_{11} \) periodicity for the alcohols due to the onset of dielectric relaxation in the 600-900 MHz region shown in Fig. 5.

**Dielectric permittivity determination**

The shape and periodic pattern of the \( S_{11} \) provides definition for fitting the modeled scatter function using eqs. (5) - (8).

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**Figure 4.** Original waveform (response function) with the Nicolson (1973) ramping algorithm applied to yield the modified waveform.

**Figure 5.** The periodic nature of the \( S_{11} \) is proportional to the dielectric permittivity of the material (e.g., water, ethanol, octanol).
Probe parameters in eqs. (5) - (7) are known or measured. The 5 coefficients of the Cole-Cole model (eq. (8)) were either assumed constant \( (\beta = 0, s_{DC} = EC_b) \) or used as fitting parameters \( (g_s, g_4, f_{rel}) \) for modeling the discrete \( S_{11} \) data. An optimization routine was established with an objective function which minimized the sum of squared differences between the transformed \( S_{11} \) and the modeled \( S_{11} \). Key fitting features in the optimization include the first and second peaks and the valley in between, which, for Octanol, comprises the entire frequency range shown in Fig 5. The static permittivity parameter was found to be most sensitive to matching these features. Further work is needed for understanding the interaction between these fitting parameters and their influence on the resulting permittivity. For example, we expect \( g_s \) to be greater than \( g_4 \) so it may be that \( g_4 \) can be expressed as a function of \( g_s \) for initial guesses of these parameters. Also, if we know the measurement frequency range of the TDR (e.g., 20 kHz to 1.5 GHz), it may be possible to estimate the average frequency content of the TDR measurement and calculate the dielectric permittivity as a function of this frequency using eq. (7). An additional benefit of the optimized parameters comes in the information on real and imaginary permittivities and relaxation effects described by the Cole-Cole parameters.

**RESULTS**

Both coaxial and 3-wire TDR probes were used to obtain permittivity measurements in saturated sand having variations in solution electrical conductivity. Fig. 6 shows the \( S_{11} \) derived from the DFT of the TDR measured waveform using a 3 cm probe \( (EC_w = 18 \text{ dS m}^{-1}) \). The optimized Cole-Cole parameters were \( g_s = 27, g_4 = 24, f_{rel} = 1.1 \text{ GHz}, s_{DC} = 0.42 \text{ S m}^{-1} \), and \( \beta \) was set to zero. Despite the increased dispersion of the data beyond about 1.5 GHz (upper frequency range for TDR), there is sufficient structure in the scatter function within the TDR frequency range for fitting the Cole-Cole model to the data. A critical feature for fitting is the first harmonic (“valley”) in the \( S_{11} \), to which additional weight can be applied for more efficient optimization. Longer probe length increases signal attenuation, which reduces the \( S_{11} \) amplitude to the point where parameter optimization ultimately fails.

The comparison of bulk permittivity determinations using time domain analysis and frequency domain analysis are illustrated in Fig. 7 where both are given as a function of solution electrical conductivity. The dielectric permittivity of the saturated Millville silt loam soil is indicated as shaded grey in the figure at a permittivity of around 27. Shorter TDR probes (e.g., < 10 cm) are unreliable in time domain analysis and results for 10 and 15 cm probes diverge for solution electrical conductivities much greater then 6 dS.
In the frequency domain analysis, permittivity determinations using longer probes fail and diverge as a function of probe length. Results using a 3 cm probe length, provide reliable permittivity values approaching 30 dS m$^{-1}$. Results from the 2 cm probe are similar, but diverge at lower EC levels than the 3 cm probe. The benefit of extending the permittivity measurement range under saline conditions comes at the sacrifice of time domain analysis capability, where shorter probes are incompatible.

**CONCLUSIONS**

Automation of time- to frequency-domain transformation of TDR waveforms facilitates measurement of bulk permittivity for water content determination in saline soils where time domain analysis fails. Enhancement of recoverable permittivity information was attainable using shorter TDR probes. Automation of this process, should provide reliable and near real-time measurement of water content under saline conditions. The Nicolson ramping algorithm provides a robust method for preparing waveforms for discrete Fourier transform. Use of an artificially generated input function in the time domain provides a convenient and rapid means of obtaining the input function and which avoids the need to remove a conductor from the TDR probe. Information on sample dielectric permittivity is preserved in the frequency domain for lossy media with electrical conductivities 4 times greater than the practical upper limit in travel time analysis of TDR waveforms.

**REFERENCES**


Friel, R., and D. Or, Frequency analysis of time-domain reflectometry (TDR) with application to dielectric spectroscopy of soil constituents, Geophysics, 64 (3), 1-12, 1999.


