ABSTRACT

Numerical models have been applied successfully to the analysis of the sensitivity and spatial sample areas of both conventional and alternative time domain reflectometry (TDR) probes. However, no similar treatment has been presented for the spatial sensitivity of TDR to lateral variations in electrical conductivity, $\sigma$. The objective of this investigation was to examine the response of conventional two- and three-rod probes to sharp changes in $\sigma$ of the media within their sample areas. Numerical analyses were performed for probes of varying rod diameters and separations for sharp interfaces both parallel to and perpendicular to the plane containing the rods. The results show clear effects of the $\sigma$ distribution on the probe response that are well described by a numerical model developed to predict the response of TDR probes to laterally heterogeneous dielectric permittivity, $K$, distributions. We show that if the pore water $\sigma$ is spatially uniform in the transverse plane then the TDR sample area for $K$ and $\sigma$ will be nearly identical for standard, uncoated parallel rod probes.
INTRODUCTION

Time domain reflectometry (TDR) is a widely accepted method for volumetric water content (θ) measurement in soils due to its ability to make rapid, nondestructive, automated measurements of θ in a wide range of soils with minimal soil-specific calibration [Topp et al., 1980]. In addition, TDR waveforms contain information that can be used to infer the electrical conductivity, σ, of the surrounding medium [Dalton et al., 1984; Dalton and Poss, 1990], allowing for solute transport monitoring in variably saturated media [e.g. Elrick et al., 1992; Kachanoski et al., 1992; Nissen et al., 1998]. The ability to monitor volumetric water content and electrolytic solute concentration with a single instrument suggests that TDR can be used to monitor solute transport during transient unsaturated flow [Persson, 1997; Risler et al., 1996]. Ferré et al. [2000b] point out potential limitations to this approach if θ varies along the TDR probes. A further potential limitation lies in the underlying assumption that θ and bulk electrical conductivity (σ_a) are measured within the same sample volume for a given probe.

Given that TDR probes are simple and inexpensive to construct and that the flexibility of their design allows for a wide range of modifications, probes can be designed to best-fit specific measurement needs. However, the probe geometry (defined by the number, size, and spacing of the rods), the presence of nonmetallic probe components within the sample area (e.g. resistive coatings), and the distribution of soil electrical properties all influence the spatial sensitivity of a given TDR probe. Knight et al. [1997] introduced the use of numerical models to define the spatial sensitivity of TDR probes based on the geometry of the probe and the distribution of dielectric permittivities in the plane transverse to the direction of electromagnetic (EM) wave propagation. Later work showed how these methods can be used to predict the response [Ferré et al., 1998] and the sensitivity distribution [Ferré et al., 2000a] of alternative TDR probe designs that incorporate nonmetallic components in their construction. Recently, the modeling approach has been used directly in the design of alternative probes, minimizing the construction and testing of prototype probes [Nissen et al., 2001]. In a companion article in these proceedings, this modeling approach is shown to describe the spatial sensitivity of TDR probes adjacent to sharp dielectric boundaries. While this numerical modeling approach has been shown to be highly effective for predicting the response of alternative TDR probes to variations in K in the plane transverse to the direction of EM wave propagation, no similar analyses have been performed to determine the spatial sensitivity of TDR to variations in the σ_a of the surrounding medium.

OBJECTIVES

The objectives of this investigation were: (i) to develop a theoretical framework for the analysis of the spatial sensitivity of TDR to variations in σ in the plane transverse to the direction of EM wave propagation; (ii) to measure the effects of sharp horizontal gradients in σ in the plane transverse to the direction of EM wave propagation on the TDR-measured σ_a for a range of two- and three-rod probes; (iii) to apply the numerical analyses developed by Knight et al. [1997] to describe the spatial sensitivity of TDR to heterogenous σ distributions in the plane transverse to the direction of EM wave propagation; and (iv) to compare the spatial sensitivity of TDR to dielectric permittivity and electrical conductivity to identify potential limitations to the simultaneous measurement of water content and solute transport with TDR.

THEORY

We present a theoretical basis for the analysis of the spatial sensitivity of TDR to variations in σ in the plane transverse to the direction of EM wave propagation under spatially uniform θ and hence K conditions. The development of this method of analysis draws on parallels to similar developments presented for the spatial sensitivity of TDR to variations in dielectric permittivity in the plane transverse to the direction of EM wave propagation. A more complete description of the latter development is presented in Knight [1992].
THE ELECTRICAL CONDUCTIVITY RESPONSE OF TDR

Dalton et al. [1984] first demonstrated that the attenuation of a TDR pulse could be used to estimate \( \sigma \) simultaneously with estimation of soil water content from the pulse speed. Dalton and van Genuchten [1986] presented an analysis of the physical principles underlying the TDR method for measuring both \( \theta \) and \( \sigma \) in porous media. We will extend their analysis to investigate the spatial sensitivity of TDR to lateral variations of \( \sigma \) for spatially uniform \( \theta \) and hence \( K \) conditions. Our development will follow that of Knight (1992) for the spatial sensitivity of TDR measurements to small variations of \( K \) in the plane transverse to the direction of EM wave propagation.

In an ideal lossless transmission line a wave or pulse propagates at constant speed without energy loss or change of shape. In lossy media, energy loss occurs if there is a series resistance in the direction of propagation, or if there is a shunt conductance through the medium between the conductors. In general the wave speed and attenuation rate are frequency dependent. If the medium between the conductors has an electrical conductivity of \( \sigma \) per unit length of material, then this will cause a shunt electrical conductivity \( G \) per unit length of transmission line between the conductors. The relationship between \( G \) and \( \sigma \) depends on the voltage distribution around the conductors, which is determined by their geometrical configuration and the voltages on the conductors. \( G \) represents a spatial average of the \( \sigma \) distribution around the probe. The spatial weighting function that describes this spatial averaging of \( \sigma \) determines the spatial sensitivity of the TDR probe to the \( \sigma \) distribution in the plane transverse to the direction of propagation.

The theory of the wave propagation along lossy transmission lines with spatially uniform parameters is well developed [e.g. Liboff and Dalman, 1985; and Smythe and Yeh, 1972]. The voltage \( V(z, t) \) and current \( I(z, t) \) on the transmission line are assumed to be functions of the time of propagation, \( t \), and the distance along the transmission line, \( z \). Then \( V(z, t) \) and \( I(z, t) \) satisfy the system of partial differential equations

\[
\frac{\partial V}{\partial z} = -RI - L \frac{\partial I}{\partial t}, \tag{1}
\]

\[
\frac{\partial I}{\partial z} = -GV - C \frac{\partial V}{\partial t},
\]

where \( L \) is inductance, \( R \) is series resistance, \( C \) is capacitance and \( G \) is shunt conductance, all expressed per unit length. These equations can be solved readily in the frequency domain if it is assumed that the solutions are periodic with time, \( t \), dependence proportional to the frequency, \( \omega \), and the imaginary variable, \( i \), as \( \exp(i\omega t) \),

\[
\frac{\partial V}{\partial z} = -RI - i\omega LI \tag{2}
\]

\[
\frac{\partial I}{\partial z} = -GV - i\omega CV
\]

Differentiating and combining these equations gives,

\[
\frac{\partial^2 V}{\partial z^2} = \lambda^2 V, \tag{3}
\]

where the complex propagation constant, \( \lambda \), satisfies
The general solution of (3) is

\[ V(z) = A \exp(-\lambda z) + B \exp(\lambda z). \]  

(5)

where \( A \) and \( B \) are constants chosen to satisfy the boundary conditions. The propagation constant, \( \lambda \), is usually written in terms of its real and imaginary parts as

\[ \lambda = \alpha + i\beta. \]  

(6)

The solution of Eq. [1] that decays in the positive \( z \) direction is then,

\[ V(z, t) = A \exp(-\alpha z) \exp(i(\omega t - \beta z)). \]  

(7)

An EM wave propagating along the transmission line in the direction of positive \( z \) is attenuated as \( \exp(-\alpha z) \) and has speed \( \omega/\beta \). In general both the attenuation rate and the wave speed are frequency dependent, giving rise to dispersion for a multiple frequency pulse. In the time domain, a pulse will change shape as the components of different frequencies move at different speeds and experience different attenuation.

If \( R \) and \( G \) are both zero then there is no energy loss and

\[ \alpha = 0, \quad \beta = \omega(LC)^{0.5} \]  

(8)

and the wave speed is \( v = (LC)^{-0.5} \) as is generally assumed in TDR analysis.

The low-loss approximation holds in the lower frequency region in which

\[ R << \omega L, \quad G << \omega C \]  

(9)

For these conditions, the propagation constant can be expanded as a Taylor series to give

\[ \alpha = \left(\frac{R}{2L} + \frac{G}{C}\right) \left(\frac{\omega}{LC}\right)^{0.5}, \quad \beta = \omega(LC)^{0.5}, \quad v = (LC)^{-0.5} \]  

(10)

Previous analyses of the \( \sigma \) response of TDR (e.g. Dalton et al., [1984]; Yanuka et al., [1988]; and Topp et al., [1988]) implicitly use the low-loss approximation. The measured or estimated value of the attenuation coefficient, \( \alpha \), is used to infer the shunt conductance, \( G \), from Eq. [10]. Then the appropriate formula for the shunt conductance of a transmission line is used to infer the bulk electrical conductivity, \( \sigma_a \).

Equation [10] shows that the wave speed for conditions satisfying the low-loss assumption is the same as the wave speed in the no-loss case. Furthermore, the wave speed and attenuation rate are independent of frequency for the lower frequency band satisfying Eq. [9]. Equation [10] shows further that the losses due to series resistance \( R \) and shunt conductance \( G \) are additive. As a result, losses due to the imaginary part of the dielectric permittivity and of the \( \sigma_a \) cannot readily be distinguished, as found by Topp et al. (2000).

**SPATIAL WEIGHTING OF PERMITTIVITY AND ELECTRICAL CONDUCTIVITY BY TDR**

The concept of a spatial weighting function was introduced by Baveye and Sposito [1984] and further explored by Cushman [1986]. Knight [1992] calculated an approximate spatial weighting function for the response of a TDR probe to small departures from uniformity of the distribution of dielectric permittivity.
\( K(x, y) \) in the lateral \((x, y)\) plane, which is oriented transverse to the direction of propagation. The voltage distribution \( V(x, y) \) in the lateral plane satisfies the equation

\[
\nabla(K \nabla V) = 0
\]

with the boundary conditions of constant voltage \( V_i \) on one conductor of a two-rod probe and - \( V_i \) on the other rod. The rod size and separation is chosen to form a representative cross section of a given probe geometry. The spatial weighting function is determined by the normalised energy distribution, which is proportional to the square of the gradient of the voltage distribution satisfying Eq. [11]. The spatial weighting function can be considered as the weighting function that the probe uses to average the spatially heterogeneous \( K(x, y) \) distribution to arrive at the transmission line capacitance \( C \) in Eq. [1]. Alternatively, the spatially-weighted \( K(x, y) \) represents the dielectric permittivity that would be measured with the modeled TDR probe.

Likewise, we can consider a system with uniform \( K \), and examine the effects of small departures from uniformity of the distribution of electrical conductivity \( \sigma(x, y) \) in the lateral plane. The voltage distribution \( V(x, y) \) in the lateral plane satisfies the equation

\[
\nabla(\sigma \nabla V) = 0
\]

with the boundary conditions of constant voltage \( V_i \) on one conductor of a two-rod probe and - \( V_i \) on the other rod. By analogy with the solution of Knight [1992], the apparent measured value of \( \sigma \) in the lateral plane then is determined by the same spatial weighting function, which is proportional to the square of the gradient of the voltage distribution. Therefore, the probe can be considered to apply the same spatial weighting function that it used to determine an equivalent \( C \) to “average” the electric conductivity distribution to arrive at the transmission line shunt conductance \( G \) in equation (1).

For example, the capacitance per unit length, \( C \), of a two-wire transmission line is [Table 5b-3, Smythe and Yeh, 1972],

\[
C = \frac{\pi \varepsilon_0 K}{\cosh^{-1}\left(\frac{s}{d}\right)}
\]

where \( \varepsilon_0 \) is the dielectric permittivity of free space, \( d \) is the diameter of each wire, and \( s \) is the separation of their axes. Likewise, the conductance per unit length, \( G \), of a two-wire transmission line is [Table 5b-3, Smythe and Yeh, 1972],

\[
G = \frac{\pi \sigma}{\cosh^{-1}\left(\frac{s}{d}\right)}
\]

Comparison of equations (13) and (14) shows the equivalence of the geometrical factors applied to the spatial averaging of dielectric permittivity \( (\varepsilon_0 K) \) and electrical conductivity \( (\sigma) \) by TDR. Based on the similarity of these transmission line responses, we propose that TDR will show identical \( \sigma \) and \( K \) responses to equivalent \( K \) and \( \sigma \) distributions in the transverse plane. Therefore, we present a modified form of the spatial weighting analysis to describe the spatial sensitivity of TDR to \( \sigma \).

**NUMERICAL ANALYSIS OF THE SPATIAL WEIGHTING OF \( \sigma \) BY TDR**

Early efforts to define the sample area of TDR probes relied on direct measurements of the change in the measured \( K \) with changes in the distribution of \( K(x, y) \) in the plane transverse to the long axis of TDR probes [Baker and Lascano, 1989]. Petersen et al. [1995] showed that this approach was able to predict the
response of TDR probes buried at shallow depths near the ground surface. No similar investigations have been presented to determine the spatial sensitivity of TDR to lateral $\sigma$ variations.

Following the development of Knight [1992] and Knight et al. [1997], we present a numerical analysis of the spatial weighting function for a given TDR probe geometry. From the preceding development, we propose that this analysis can be used to predict the $\sigma_C$ response of TDR probes of any geometry to any spatial distribution of $\sigma$ in the plane transverse to wave propagation. Full development of the theory underlying the numerical analysis is provided in Knight et al. [1997]. The approach is based on a finite element solution of the two-dimensional Laplace equation in the transverse plane. Rods are internal boundaries to the domain and are set to constant potentials. The domain is chosen to be sufficiently large that a zero potential and a zero flux boundary condition give the same result. The potential distribution within the domain, $\Phi(x, y)$, is determined for a given $\sigma(x, y)$ distribution. The spatial weighting function, $w(x, y)$, is defined for each element within the domain based on the square of the gradient of the potential across that element.

$$w(x, y) = \frac{\left| \nabla \Phi(x, y) \right|^2}{\iint_{\Omega} \left| \nabla \Phi_0(x, y) \right|^2 dA}$$

where $\Phi_0(x, y)$ is the potential in a given element for a homogeneous $\sigma$ distribution, $\Omega$ is the domain of integration, and $dA$ is the area of the element. The predicted measured $\sigma_a$ is then,

$$\sigma_a = \iint_{\Omega} \sigma(x, y)w(x, y)dA$$

The spatial weighting function describes the sensitivity of the instrument to any given point in the domain. The weighting function is defined such that,

$$\iint_{\Omega} w(x, y)dA = 1$$

The equivalent $\sigma$ can also be determined for the entire domain based solely on the total electrical flux into or out of the interior constant potential boundaries. In other words, the flux into the domain is a function of the geometry of the system, the boundary conditions, and the $\sigma(x, y)$ distribution. The flux out of the domain can be calculated for a uniform medium as $q_0$, with the electrical conductivity set to a constant value of $\sigma_0$. The flux out of the domain can be calculated for any electrical conductivity distribution, $\sigma(x, y)$ as $q$. Given that the domain geometry and the boundary conditions are unchanged, the effective electrical conductivity of the heterogeneous system is defined as,

$$\sigma_a = \frac{q_0}{q\sigma_0}$$

**EXPERIMENTAL AND NUMERICAL METHODS**

A box was constructed to allow for the controlled advance of a sharp fluid boundary into and through the sample volume of TDR probes (Figure 1).
The box was constructed of a PVC base and four PVC walls. The inner dimensions of the completed box were 0.3 x 0.3 x 0.3 m. Two opposite walls included a transparent PVC window to allow for visual location of a liquid-air interface within the box. The remaining two opposite walls included a circular inset with five NPT threaded holes capable of accommodating Swagelok male connector tube fittings. Using these fittings, rods of diameters equal to 3.2, 4.6 and 6.4 mm can be inserted through the box walls. The effective length of the probe rods inside the box is approximately 285 mm. Two and three-rod probes can be formed with outer rod spacings equal to 17.5, 35.0, 52.5, and 70.0 mm for the two-rod probe and 35.0 and 70.0 mm for the three-rod probe, respectively. The circular insets can rotate to six different positions, allowing the probe axis passing through the center of the probe rods to lie in the horizontal plane, the vertical plane, or 45° from the horizontal. For these experiments, we used probes formed by placing horizontal rods in the horizontal plane (referred to as horizontal probes) and horizontal rods placed in the vertical plane (referred to as vertical probes). Figure 2 shows a representative cross section through a vertical two-rod probe and through a horizontal three-rod probe.

Figure 2: Schematic diagram of a three-rod horizontal probe (left) and a two-rod vertical probe (right). The horizontal line shows the height of the midpoint of the probes. All measurements are related to this midpoint.

The box was placed on a Sartorius IC 34000 P balance with 0.5 g precision, which was interrogated every 19 seconds. A Tektronix 1502B cable tester was connected to the TDR probe and a waveform was collected every 19 seconds using TACQ software from Dynamax. Demineralized water was used for
testing. To alter $\sigma$ of the water, KCl was added to 0.001M and 0.005 M. Each solution was tested separately. Solution was added slowly at the bottom of the box through a vertical tube. The distance between the liquid-air interface was determined from the change in mass, the fluid density added and the cross sectional area of the box. This was checked visually at the time at which the interface reached the bottom and top of the probe rods.

A finite element grid was constructed for each probe geometry. For this investigation the model was defined for 13 probe/fluid combinations, as described in Table 1. For each simulation, the domain was divided into 60 layers to model a series of liquid-air interfaces. However, in most cases some of the layers were pooled in areas of low sensitivity to reduce the modeling task. Approximately 16000 elements were used to discretize the domain. The run time for each liquid-air interface location was approximately 1 minute.

<table>
<thead>
<tr>
<th>Legend</th>
<th>Number of Rods</th>
<th>Rod Diam. [mm]</th>
<th>Rod Sep. [mm]</th>
<th>Fluid</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>6.35</td>
<td>70</td>
<td>Dem. water</td>
<td>Horizontal</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>6.35</td>
<td>70</td>
<td>0.001 KCl</td>
<td>Horizontal</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>6.35</td>
<td>70</td>
<td>0.005 KCl</td>
<td>Horizontal</td>
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<tr>
<td>D</td>
<td>2</td>
<td>6.35</td>
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<td>E</td>
<td>2</td>
<td>6.35</td>
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<td>0.001 KCl</td>
<td>Vertical</td>
</tr>
<tr>
<td>F1</td>
<td>2</td>
<td>6.35</td>
<td>70</td>
<td>0.005 KCl</td>
<td>Vertical</td>
</tr>
<tr>
<td>F2</td>
<td>2</td>
<td>6.35</td>
<td>70</td>
<td>0.005 KCl</td>
<td>Vertical</td>
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<tr>
<td>G</td>
<td>3</td>
<td>6.35</td>
<td>70</td>
<td>Dem. water</td>
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<td>H</td>
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<td>K</td>
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<td>3</td>
<td>6.35</td>
<td>70</td>
<td>0.005 KCl</td>
<td>Vertical</td>
</tr>
</tbody>
</table>

Table 1: Legend describing the rod geometries and fluids used for all 13 experiments. Note that for experiment F1 the shield was connected to the lower rod of a two-rod probe and the central conductor was connected to the upper rod; for experiment F2 these connections were reversed.

RESULTS AND DISCUSSION

The experimental and numerical results will be presented in two sections: directly measured influences of sharp electrical conductivity boundaries on probe responses, and comparisons of the measured and predicted probe responses.

MEASURED PROBE RESPONSES

The electrical conductivities determined directly from TDR measured signal loss are shown as a function of the depth of water in the box in Figure 3. The center height of the top and bottom rods are shown for vertical configurations. The center height of the probe is shown for all configurations. Each data series represents the measured $\sigma$ at a series of interface heights for a single probe geometry. The data series label refers to Table 1. The probe geometry is shown graphically above the legend as a representative cross section of the probe.

For each probe geometry there is a clear distinction between the $\sigma$ responses when they are immersed in air, 0.001M KCl and 0.005 M KCl. However, there is very little difference between the $\sigma$ response in air and in demineralized water. The two- and three-rod horizontal probes show a sharp increase in $\sigma$ when the fluid makes contact with the lowest edge of the rods. The probe response is more complex for the vertical probe orientations. For the two-rod vertical probe, the observed sharp rise in $\sigma$ does not occur until the interface reaches the lowest edge of the uppermost rod. The response of the three-rod vertical probe shows
two distinct regions. The first rise in measured $\sigma$ occurs when the fluid interface contacts the lowest edge of the center rod. A second increase occurs when the interface contacts the base of the uppermost rod. Although the waveforms between these discontinuities show multiple peaks, making automated travel time analysis difficult, there was no difficulty in identifying the final amplitude from the waveforms. Therefore, the $\sigma$ response is well behaved for all interface locations.

Figure 3: Measured electrical conductivity ($\sigma$) as a function of the depth of solution in the box. Three fluids were used: demineralized water (circles), 0.001M KCl (diamonds) and 0.005M KCl (squares). Four probe geometries were tested, as shown schematically as a representative cross section above each legend. The probe/fluid combinations are referenced in the legend to Table 1. For vertical geometries, the midheights of the rods are shown as vertical lines. The elevation of the center of all of the rods is shown as a single line for the horizontal configurations.

COMPARISON OF PREDICTED AND MEASURED PROBE RESPONSES

To compare the predicted response with the measured $\sigma$ and K responses, all responses and predictions were normalized using the values corresponding to measurements in air and the advancing fluid. The normalized measured dielectric permittivities for the 0.001M KCl are shown as squares in Figures 4 and 5; the 0.005M KCl solution results are shown as triangles. There was no significant difference in the $\sigma$ of air and demineralized water; therefore, the normalization led to large relative errors in the measurements. The solid lines represent the model-predicted cumulative weighting function summed from the base of the box to a given height. Discussions will be grouped as follows: (i) two- and three-rod horizontal probes (B, C, H, I), (ii) two-rod vertical probes (E, F1, F2), and (iii) three-rod vertical probes (K, L).

HORIZONTAL PROBES

The model and all of the measurements (B, C, H, I) show a response that is skewed about the height of the middle of the probe with a sharper response as the interface approaches the probe midpoint from below followed by a slower response as the interface continues to rise. This indicates that both the $\sigma$ and the K
responses are more sensitive to the medium with a lower $K$ or $\sigma$ when the interface is between the rods. The three-rod probe response is sharper than the two-rod response.

Figure 4: Normalized measured dielectric permittivity, $K$, (squares) and electrical conductivity, $\sigma$, (triangles) and model predicted cumulative weighting functions (line) as a function of the depth of solution. The probe center height is shown as a vertical line. Probe geometries and fluid compositions are referenced to Table 1.
Figure 5: Normalized measured dielectric permittivity, $K$, (squares) and electrical conductivity, $\sigma$, (triangles) and model predicted cumulative weighting functions (line) as a function of the depth of solution. The uppermost and lowermost rod center heights are shown as vertical lines for all probes. A vertical line also indicates the center height of the middle rod for the three-rod probe. Probe geometries and fluid compositions are referenced to Table 1.
For the two-rod cases (B and C) the modeled cumulative weighting function shows better agreement with the measured $K$ than with the measured $\sigma$. For the three-rod cases (H and I) there is very little difference among the measured and modeled responses. This agreement demonstrates that the spatial weighting approach based on the energy distribution describes the spatial sensitivity of TDR to both the dielectric permittivity and the electrical conductivity even in the presence of sharp contrasts in $K$ and $\sigma$ in the transverse plane.

**TWO-ROD VERTICAL PROBES**

The measured $K$ and $\sigma$ both show very good agreement with the model predictions for this probe geometry. The probe showed very little response until the interface reached the base of the upper rod. Furthermore, the polarity of the probe (connection of either the upper or lower to the shield) had no effect on either the $K$ or the $\sigma$ response, demonstrating that two-rod probes can be used without a balun with no effect on their response or sample area.

**THREE-ROD VERTICAL PROBES**

As shown in a companion article in these proceedings, the measured $K$ response of vertical three-rod probes shows poor agreement with the model predictions when the interface is located between the middle and uppermost probes. They conclude that this is due to difficulties in interpreting the complex waveforms that arise with the interface in these locations. This conclusion is supported by the excellent agreement between the model prediction and the measured $\sigma$ response. The preceding discussion demonstrates that the $K$ and $\sigma$ sample areas of TDR probes are nearly identical for similar $K$ and $\sigma$ distributions in the plane transverse to the direction of EM wave propagation. This suggests further that the $K$ response should follow the model prediction for the three-rod vertical case, as does the $\sigma$ response, and that the disagreement is due to errors in waveform interpretation.

**CONCLUSIONS**

The measured probe responses demonstrate that the $\sigma$ of a high conductivity medium cannot be measured without making physical contact with that medium. However, $\sigma$ of a low conductivity medium can be measured through a high conducting medium. One result of this is that air gaps around probes may have a more pronounced effect on the $\sigma$ response than on the $K$ response.

There is good agreement between the normalized measured $\sigma$ and the model-predicted cumulative weighting function. The only exception is the two-rod horizontal geometry. For some cases, such as three-rod vertical probes, the $\sigma$ response shows better agreement with the model predictions than does the $K$ response. This general agreement demonstrates that the spatial weighting approach developed to predict the response of TDR probes to laterally heterogeneous dielectric permittivities can be used to predict the $\sigma$ response as well. Furthermore, this indicates that the spatial sensitivity of TDR to $K$ and $\sigma$ are very similar for similar $K$ and $\sigma$ distributions in the plane transverse to the direction of EM wave propagation. Given that both $K$ and $\sigma$ show a second order dependence on $\theta$, this suggests that the $K$ and $\sigma$ sample areas may be very similar even if $\theta$ varies in the transverse plane. However, if the pore water $\sigma$ varies laterally, these sample areas will be different.
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